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LINEAR TIME-INVARIANT SPACE-VARIANT FILTERS AND THE WKB APPROXIMATION

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ABSTRACT

Computer simulated output electrical signals, based on derived mathematical models, were generated at each element in a receive planar array of complex weighted point sources. The output electrical signals depend on the frequency spectrum of the transmitted electrical signal, the far-field beam patterns of the transmit and receive planar arrays, and the time-invariant, space-variant, random transfer function of the ocean volume which was derived using the WKB approximation. The computer simulated output signals were processed by a three-dimensional FFT beamformer in order to study pulse propagation in a random medium with applications to underwater acoustic communication, target detection, and target localization. Preliminary results are reported in this paper.

INTRODUCTION

Wave propagation in a random, inhomogeneous ocean is treated as transmission through a linear, time-variant, space-variant, random communication channel [1-6]. Using the method of separation of variables and the WKB approximation, Ziomek [7] derived a time-invariant, space-variant, random transfer function of the ocean volume between transmit and receive arrays. The transfer function was time-invariant because motion was not considered. The ocean volume was characterized by a one-dimensional random index of refraction (sound-speed profile) which was a function of depth. The index of refraction was decomposed into a deterministic component and a zero mean random component. Using the transfer function, Ziomek [8] also derived a mathematical expression for the random output electrical signal at each element in a receive planar array of complex weighted point sources. The output electrical signals are expressed in terms of the frequency spectrum of the transmitted electrical signal, the far-field beam patterns of the transmit and receive planar arrays, and the random transfer function of the ocean volume. In addition, Ziomek [9] derived a coherence function, that is, the autocorrelation function of the transfer function, incorporating the WKB approximation and a transfer function, output electrical signals, and a coherence function incorporating the parabolic equation (PE) approximation [10,11]. The PE results [10,11] are based on using a deterministic component of the index of refraction equal to unity along with a three-dimensional random component.

Computer programming of the equation for the output electrical signals based on the WKB approximation has been accomplished by Vos [12]. Computer simulated output signals were generated and then processed using a three-dimensional FFT beamformer [13] in order to study pulse propagation in a random medium with applications to underwater acoustic communication, target detection, and target localization. Preliminary results obtained by Vos [12] are reported in this paper. In all cases, the transmitted electrical signal was represented by a finite Fourier series. Although the equations derived by Ziomek [7-9] allow for an arbitrary random index of refraction as a function of depth, the computer results presented here are based on using a constant gradient sound-speed profile to model the deterministic component of the index of refraction. Also, the random component of the index of refraction was assumed to be a zero mean, Gaussian, random variable that was not a function of depth and was modeled by a random number generator. The constant standard deviation σ of the random component of the index of refraction was chosen to be on the order of 10^{-4} [14,15].

COMPUTER SIMULATION RESULTS

Figures 1 and 2 represent the angular spectrum output from the three-dimensional FFT beamformer for the two test cases, case INHMG and case RINHMG, respectively. Case INHMG refers to a deterministic ($\sigma = 0$) inhomogeneous medium with a constant gradient ($g = 0.017 \text{ sec}^{-1}$) sound-speed profile. Case RINHMG is the same as case INHMG except that a zero mean, Gaussian random component of the index of refraction has been added ($\sigma = 0.25 \times 10^{-4}$). The random component was not a function of depth.

Let us next discuss the important parameters that are common to both test cases with X, Y, and Z referring to cross-range, depth, and range, respectively. The transmitted electrical signal was baseband, composed of 5 harmonics with fundamental frequency $f_0 = 1 \text{ kHz}$ and fundamental period $T_0 = 1 \text{ msec}$. The Fourier series coefficients used were $c_0 = 0$, that is, no DC component, and $c_k = 1$; $k = 1, 2, \dots, 5$. A 11×11 element transmit planar array lying parallel to the XY plane was used with rectangular amplitude weighting in both the X and Y directions. The center of the transmit array was located at $x_0 = 0 \text{ m}$, $y_0 = 1000 \text{ m}$, and $z_0 = 0 \text{ m}$. The speed of sound used at the center of the transmit array was $c_0 = 1475 \text{ m/sec}$. The frequency used to calculate the phase weights to steer the transmit beam pattern toward the center of the

5.9.1

receive array was 3 kHz. A 5x5 element receive planar array lying parallel to the XY plane was used with rectangular amplitude weighting (no phase weighting) in both the X and Y directions. The center of the receive array was located at $x_R = 200$ m, $y_R = 2000$ m, and $z_R = 1720.464$ m. One-half wavelength interelement spacings (using the minimum wavelength) were used in both the transmit and receive arrays to avoid grating lobes. Finally, 11 time samples per $T_0 = 1$ msec interval per element in the receive array were taken and processed by the beamformer.

Figures 1a through 1e demonstrate that as the frequency increases, the peak of the angular spectrum becomes more defined as would be expected since the beamwidth of the receive beam pattern decreases as the frequency increases. Referring to Fig. 1c ($f = 3$ kHz), the peak occurs at $u_0 = 0.1$ and $v_0 = 0.49$ where u_0 and v_0 are the direction cosines measured with respect to the positive X and Y axes, respectively, of the receive array. These values are estimates of the direction cosine values corresponding to the line of sight measured from the center of the transmit array to the center of the receive array. The true values are $u = 0.1$ and $v = 0.5$. Thus, there is a slight error in the estimate of v as would be expected because of ray curvature due to the speed of sound being a function of depth y . There is no error in the estimate of u since the medium is homogeneous in the X direction.

Figures 2a through 2e, when compared to Figs. 1a through 1e, respectively, show that the effects of the random component of the index of refraction is to distort the angular spectrum by enlarging the sidelobe structure. The distortion becomes more apparent at the higher frequencies. Referring to Fig. 2c ($f = 3$ kHz), the peak occurs at $u_0 = 0.1$ and $v_0 = 0.53$, which are estimates of the true values $u = 0.1$ and $v = 0.5$. The error in the estimate of v is larger for this case compared to Fig. 1c because of the scattering due to the random component of the index of refraction.

ACKNOWLEDGMENT

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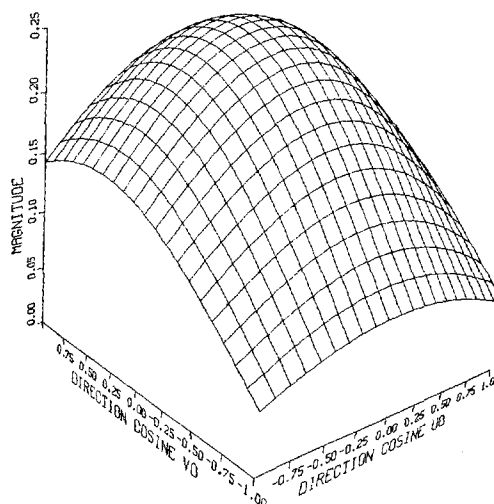


Fig. 1a. Case INHMG for $f = 1$ kHz.

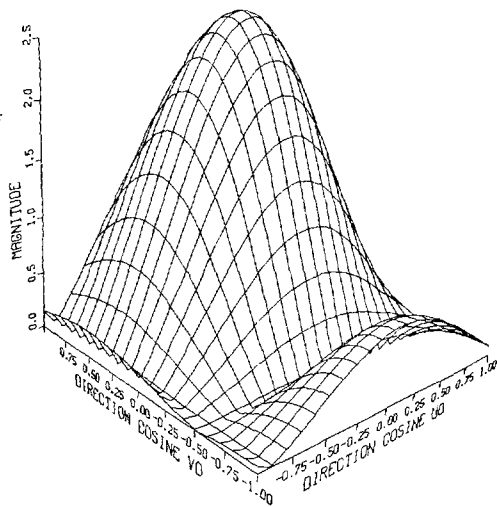


Fig. 1b. Case INHMG for $f = 2$ kHz.

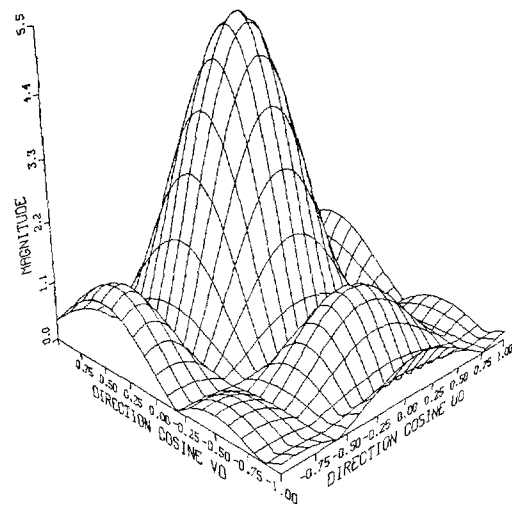


Fig. 1c. Case INHMG for $f = 3$ kHz.

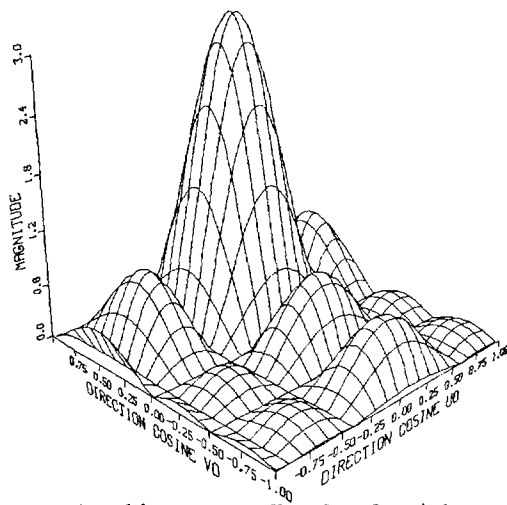


Fig. 1d. Case INHMG for $f = 4$ kHz.

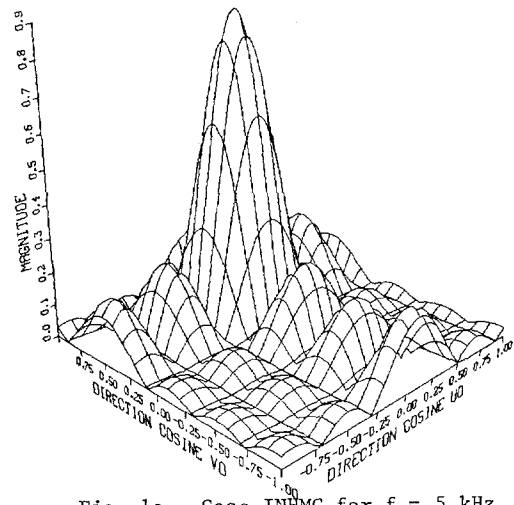


Fig. 1e. Case INHMG for $f = 5$ kHz.

Fig. 1. Angular spectrum output from the three-dimensional FFT beamformer for each of the five harmonics for test case INHMG.

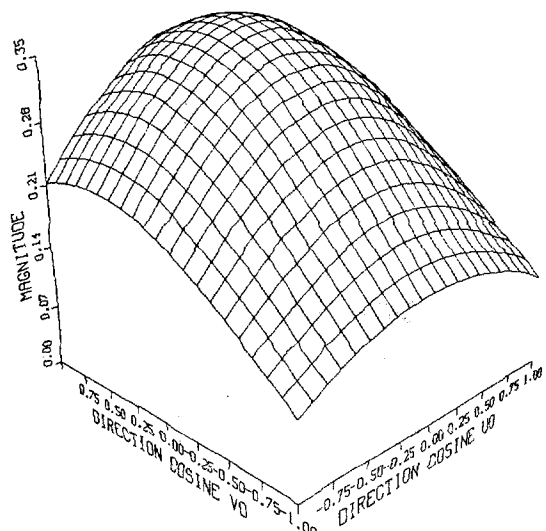


Fig. 2a. Case RINHMG for $f = 1$ kHz.

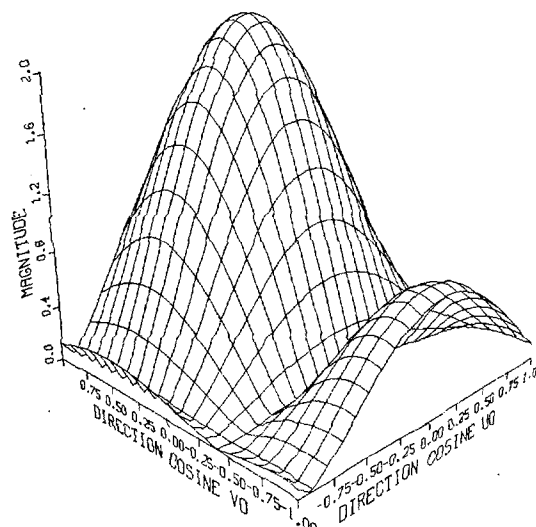


Fig. 2b. Case RINHMG for $f = 2$ kHz.

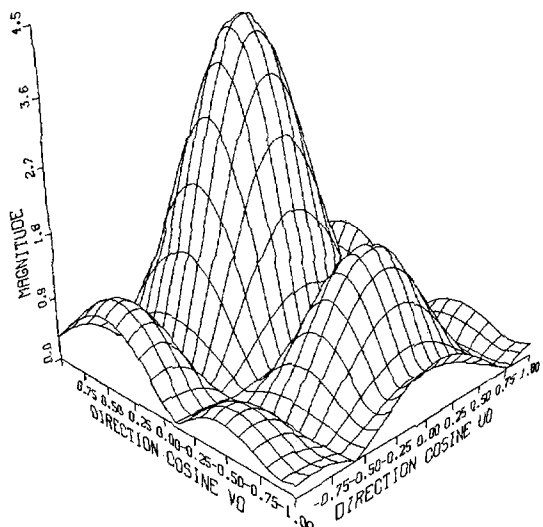


Fig. 2c. Case RINHMG for $f = 3$ kHz.

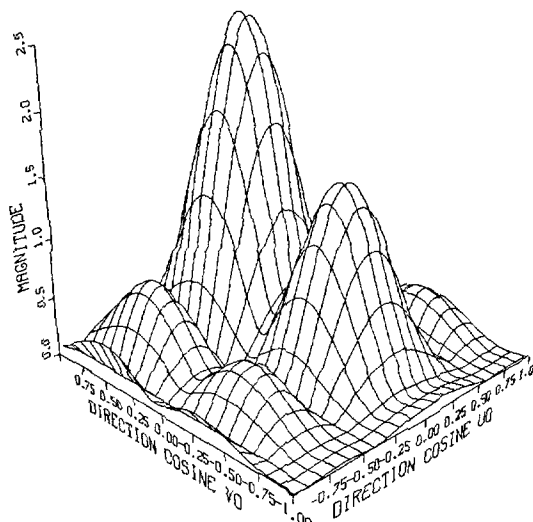


Fig. 2d. Case RINHMG for $f = 4$ kHz.

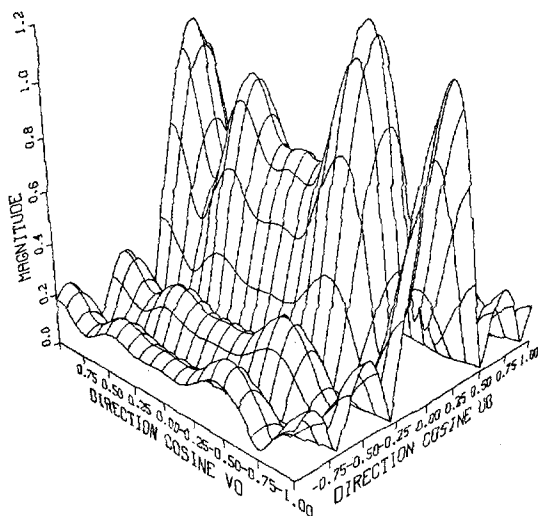


Fig. 2e. Case RINHMG for $f = 5$ kHz.

Fig. 2. Angular spectrum output from the three-dimensional FFT beamformer for each of the five harmonics for test case RINHMG.